Short Papers

An Inhomogeneous Two-Dimensional Model for the Analysis of Microstrip Discontinuities

Wojciech K. Gwarek and Cezary Mroczkowski

Abstract—**The paper proposes a new approach to two-dimensional modeling of microstrip circuits of arbitrary shape. A new model was developed to be used for finite-difference time-domain (FDTD) analysis. A characteristic feature of the new model is its inhomogeneity, i.e., the dependence of the parameters at a particular point in the two-dimensional space on the distance of that point from the strip's edge. Examples of FDTD analysis based on the new model are shown.**

I. Introduction

Analysis of microstrip discontinuities has been the focus of researchers for many years and has resulted in an enormous number of publications [1]. Different mathematical models have been used. The choice of the model results from contradicting requirements for high accuracy and small computing effort. Most commercial programs use one-dimensional models consisting of segments of transmission lines and lumped elements. Their use is, however restricted, (see, for example, [2]). The three-dimensional models use Maxwell's equations in three-dimensional space. Although significant progress in applying such models can be noted [3], [4], [12], the analysis of arbitrarily shaped circuits requires very long computing times. In the author's opinion, another generation of computer hardware and software will be required before they become commonly used tools in microwave engineering.

The two-dimensional models use Maxwell's equations in two-dimensional space. They significantly improve the accuracy of analysis when compared with one-dimensional models and require much less computer time than three-dimensional ones. Let us consider the finite-difference time-domain (FDTD) method in two and three dimensions. The change from two to three dimensions not only requires a larger number of mesh points owing to discretization in the third dimension but also increases the number of field components and makes it more difficult to define boundary conditions on curved boundaries and to model lattice terminations. These factors combined usually make the computing time for three-dimensional FDTD larger by a factor of a few hundred if not a few thousand.

Two-dimensional models were employed with different numerical methods by D'Inzeo *et al.* [5], Okoshi [7], and Menzel and Wolff [6]. Recent progress in the development of the FDTD method for arbitrarily shaped two-dimensional circuits opens the way for new classes of 2-D inhomogeneous models. One such model was proposed in [8]. It gave good results for certain types of circuits but was found to be imprecise in other types. The present paper proposes a new model which has been found to be more accurate and versatile then the previous one.

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II. Two-Dimensional Inhomogeneous Model

Let us consider a microstrip line of width w build on a substrate of permittivity ϵ_d (Fig. 1) which is characterized by its impedance, $Z(w)$, and effective permittivity, $\epsilon_{\text{eff}}(w)$, or by the unit capacitance, $C(w)$, and unit inductance, $L(w)$:

$$Z(w) = \sqrt{\frac{L(w)}{C(w)}} \tag{1}$$

$$\epsilon_{\text{eff}}(w) = v^2 L(w) C(w). \tag{2}$$

The parameters $Z(w)$ and $\epsilon_{\text{eff}}(w)$ in (1) and (2) can be evaluated by using Hammerstad–Jensen formulas [11].

Let us consider half of the microstrip from one side of the axis of symmetry. If we assume further that an increase in the width of the strip in the center of the line does not change the electromagnetic field near the open edge of the line, then we can assign to each point of the strip two functions representing the "density" of capacitance, $c(r)$, and the "density" of inductance, $l(r)$, where

$$C(w) = \int_0^w c(r)\,dr \tag{3}$$

$$1/L(w) = \int_0^w 1/l(r)\,dr. \tag{4}$$

These densities can be obtained from the known functions $C(w)$ and $L(w)$ using the formulas

$$c(w/2) = \frac{dC(w)}{dw} \tag{5}$$

$$1/l(w/2) = \frac{d(1/L(w))}{dw}. \tag{6}$$

On the basis of (5) and (6) we can define a microstrip model with inhomogeneous medium under the strip:

$$\epsilon_1(r) = c(r)h \tag{7}$$

$$1/\mu_1(r) = \frac{h}{l(r)}. \tag{8}$$

Shapes of the functions $\epsilon_1(r)$ and $\mu_1(r)$ (normalized to ϵ_d and μ_0) for a line on a dielectric substrate of $\epsilon_d = 10$ are shown in Fig. 2. They are plotted for $r > r_1$, where r_1 is assumed to be equal $0.1h$. For values of r approaching 0, direct application of (7) and (8) would lead to infinite growth of $c(w)$ and $l(w)$ owing to the unrealistic assumption that fringing fields are concentrated near the edge of the strip. This would cause numerical problems and would decrease the accuracy. Taking this into

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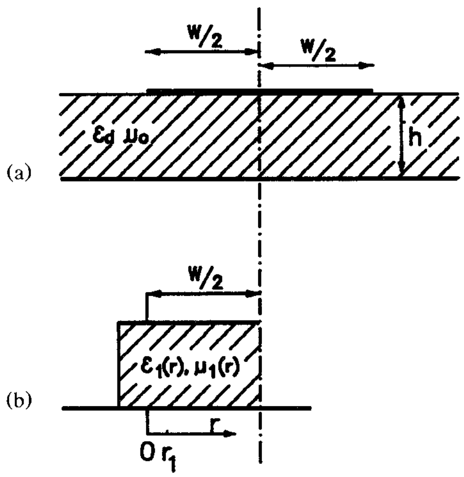

Fig. 1. (a) Cross section of a microstrip line. (b) Model of the line of (a).

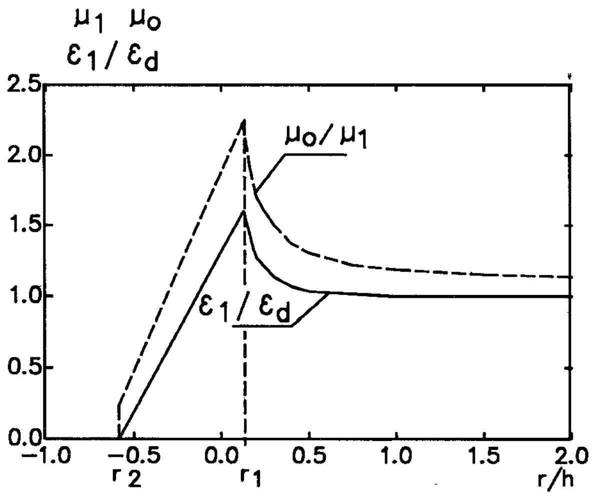

Fig. 2. Examples of the functions ϵ_1/ϵ_d and μ_0/μ_1 for a line on a dielectric substrate of permittivity ϵ_d.

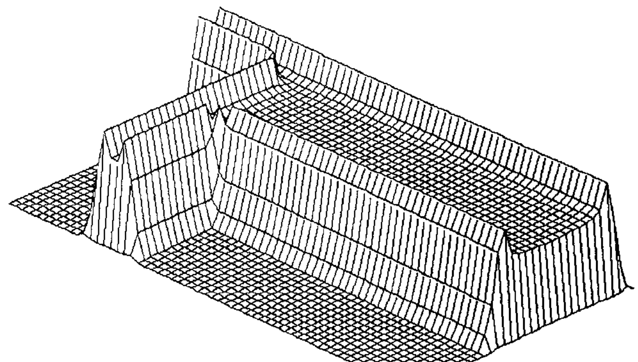

Fig. 3. Inhomogeneous model of the analyzed circuit.

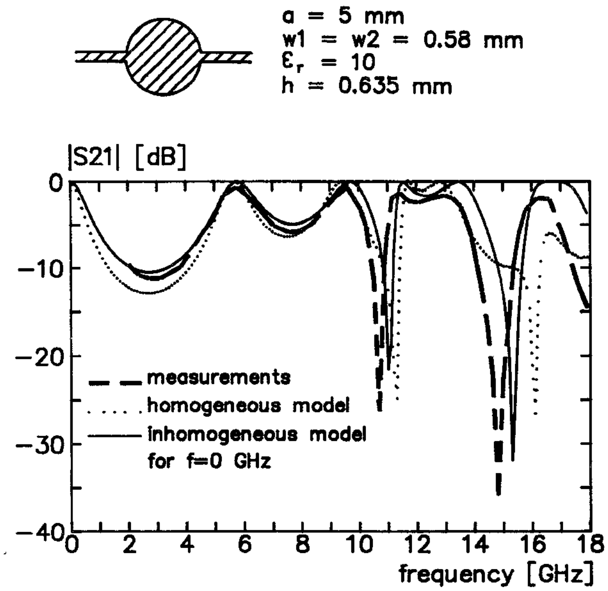

Fig. 4. Comparison of calculated and measured values of $|S_{21}|$ versus frequency for a circular microstrip circuit.

account, we have modified the functions ϵ_1 and μ_1 for $r < r_1$. The modified functions (shown in Fig. 2) are not too far from a physical interpretation and are simple enough for effective numerical application. The unit capacitance, C_0, of the line of width $w = 2r_1$ is assumed to come from a linear distribution of ϵ_1 in the range of r from r_1 to

$$r_2 = r_1 - 2C_0 h / \epsilon_1(r_1). \tag{9}$$

Thus the edge of the model (where the magnetic wall is assumed) is moved with respect to the edge of the real circuit by the distance $|r_2|$. The function of $\mu_1(r)$ is assumed to be of the same shape as in Fig. 2, with the integral of $1/\mu_1$ between r_1 and r_2 corresponding to $1/L(r_1)$.

In the two-dimensional inhomogeneous model we assume that the permittivity and permeability of the medium filling the model at a particular place in the circuit are equal to $\mu(x, y) = \mu_1(r)$ and $\epsilon(x, y) = \epsilon_1(r)$, where r is the distance of the point (x, y) from the nearest edge of the circuit. The two-dimensional distribution of $\epsilon(x, y)$ of the circuit analyzed further in Example 2 is shown in Fig. 3.

The newly developed model was implemented in the Quick-Wave [10] programming package and checked in FDTD calculations of various microstrip circuits. Three of them will be discussed here as examples.

Example 1

We consider a circular microstrip resonator of radius 5 mm connected to a 50 Ω input and output line with the lines making an angle of 180°. The substrate is alumina of $\epsilon = 10$ and $h = 0.635$ mm. This example has been considered in [5], from which the results of measurements are taken. Let us first assume a homogeneous model of the microstrip resonator. The results of FDTD calculations seen in Fig. 4 show large discrepancies with the results of measurement for higher frequencies. As explained in [5], such discrepancies come from the fact that in a homogeneous resonator the resonant frequency of mode 41 (corresponding to mode TM_{410} in a cylindrical resonator with open sidewalls) is below the frequency of mode 12, whereas it has been proved by experiment that in a microstrip resonator the second one appears below the first. In the homogeneous model used in time-domain calculations, it is impossible to produce this shift of frequencies. On the contrary our inhomogeneous 2-D model apparently causes a proper shift of the resonant frequencies, which is visible from the results of calcula-

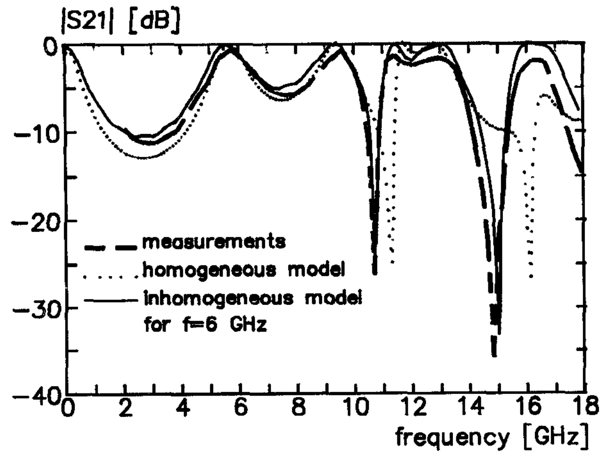

Fig. 5. Modification of the results of the calculation of Fig. 4 under the assumption of an inhomogeneous model for $f = 6$ GHz.

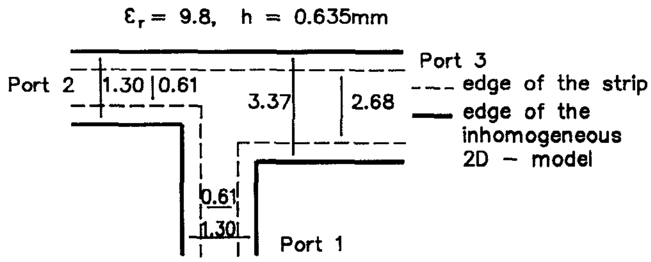

Fig. 6. A microstrip T junction on alumina substrate.

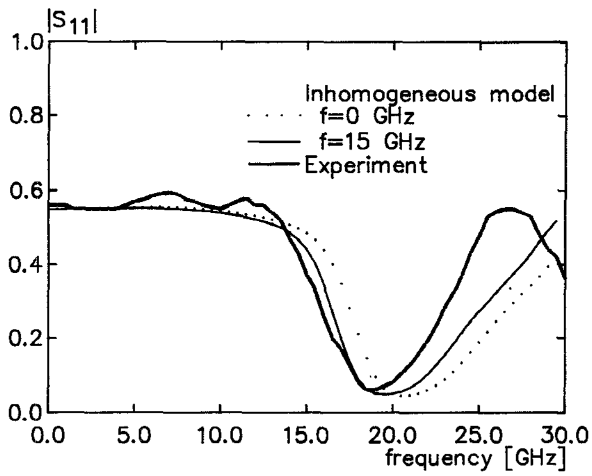

Fig. 7. $|S_{11}|$ of the junction of Fig. 6.

tions presented also in Fig. 4. These calculations were obtained using the version of the FDTD method described in [8] and were used in the Quick-Wave program [10]. The unit parameters $L(w)$ and $C(w)$ used for calculations were taken from the closed-form formulas of [1, ch. 3.3.] and [11] for the frequency $f = 0$. It can be seen in Fig. 4 that the calculated higher resonant frequencies are shifted up with respect to those obtained from measurements. The discrepancy is due to microstrip dispersion, which may be taken into account to some extent by assuming the parameters $L(w)$ and $C(w)$ for higher frequencies. In the case considered, after assuming $L(w)$ and $C(w)$ for $f = 6$ GHz, we have obtained results very close to measurements also for higher frequencies (Fig. 5).

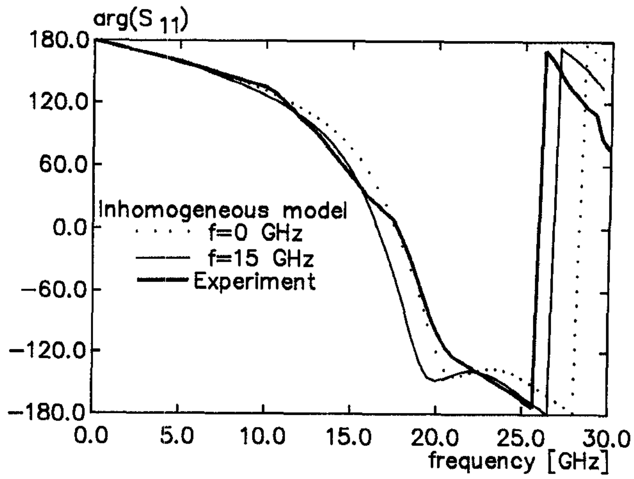

Fig. 8. $\arg(S_{11})$ of the junction of Fig. 6.

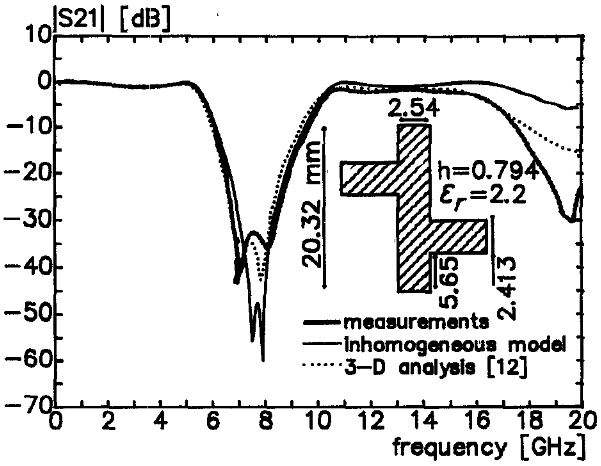

Fig. 9. $|S_{21}|$ of a microstrip low-pass filter.

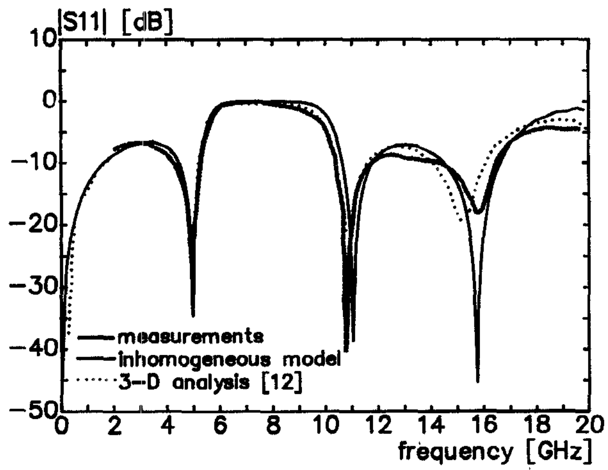

Fig. 10. $|S_{11}|$ of the microstrip low-pass filter of Fig. 9.

Example 2

A microstrip T junction of one 20 Ω line and two 50 Ω lines is considered. Its shape is shown in Fig. 6. The substrate is alumina of $\epsilon = 9.8$ and height $h = 0.635$ mm. Figs. 7 and 8 show the results of calculations compared with the results of measurements taken by Gronau and Wolff at the University of Duisburg [9]. In this example we took values of $L(w)$ and $C(w)$ for $f = 0$ GHz and $f = 15$ GHz (Figs. 7 and 8).

Very good agreement between the measurements and calculations was also obtained for many other T junctions of different shapes, which are not presented here.

Example 3

We consider a microstrip low-pass filter which has been measured and analyzed by a three-dimensional FDTD method by the authors of [12]. The substrate has $\epsilon_r = 2.2$ and $h = 0.794$ mm. In Figs. 9. and 10 we compare the results published in [12] with those obtained using our 2-D inhomogeneous model. Good agreement again is obtained. Certain discrepancies at high frequencies are most probably due to radiation. Our analysis took about 7 min on a PC-386 working under DOS while the 3-D analysis [12] of the same example was reported to take 8 h on a VAX station 3500.

III. Conclusions

The paper has presented a new two-dimensional model for the analysis of arbitrarily shaped microstrip circuits. The model was checked in the FDTD program prepared by the authors to run on a PC. It was found very useful for investigating new designs of junctions, resonators, and patch couplers. However, it must be admitted that the model is effective only in cases where phenomena that are typically three-dimensional, such as radiation and coupling, can be neglected.

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Accurate Formulas for Efficient Calculation of the Characteristic Impedance of Microstrip Line

K. K. M. Cheng and J. K. A. Everard

Abstract —**A numerically efficient and accurate method for the derivation of the characteristic impedance of an open microstrip line assuming the quasi-TEM mode of propagation is presented. It is based on the spectral-domain method incorporating functions of rectangular shape for describing the surface charge density distribution on the conductor strip. This gives rise to integrals which can be analytically evaluated. The formulas thus obtained can readily be implemented on a desktop computer. It is found that the discrepancies between the results derived from the proposed method ($N = 3$) and from the substrip method are less than 0.26% through a wide range of w/h ratios and relative permittivity values.**

I. Introduction

A vast amount of literature [1]–[7], [9]–[11] has been published on the numerical computation of the characteristic impedance of microstrip. Wheeler employed an approximate conformal mapping method in the study of microstrip in a mixed dielectric media [2]. Silvester and Farrar [3], [4] treated this problem by the method of moments and dielectric Green's function. Poh *et al.* [5] applied the spectral-domain method to the analysis of microstrip and showed that by careful treatment of the edge singularities of the charge density on the strip, the method can often give rise to accurate results with only a few basis functions. In this paper, a new method based on the spectral-domain approach is presented for determining the characteristic impedance of an open microstrip line. By selecting the rectangular shaped functions as the basis functions, the resulting integrals in the solution can be efficiently evaluated. Furthermore, the number of basis functions required is minimized by searching for the optimum widths of these rectangular shaped functions, which will give the least calculated impedance error. The proposed method is therefore numerically efficient, easy to implement, and highly accurate. For purposes of comparison, results calculated by the substrip method and by our formulas are shown. The extension of this method to the modeling of microstrip with thick strip conductor and covered microstrip is discussed.

II. Method of Analysis

The study of microstrip is carried out under the assumptions that the mode of propagation is quasi-TEM and the line has negligible loss. In this case the characteristic impedance, Z_0, of a microstrip line is given by

$$Z_0 = \frac{1}{v\sqrt{CC_0}} \tag{1}$$

where v is the velocity of light in vacuum, C is the capacitance per unit length of the microstrip shown in Fig. 1, and C_0 is the

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